

Typical mistakes students make in mathematics - reasons and possible ways out

Problems in mathematics

Is it impossible to understand mathematics?

Sometimes this question may occur, because we face the following problems in math education:

- Many students have severe problems in math.
- Year by year similar or even the same mistakes are made.
- Some mistakes seem to be difficult to remove.
- Reasons for the dilemma:
Deficits of fundamental abilities and understanding of concepts

What are the mistakes we have to deal with?

The mistakes can be divided into a few categories:

1. Fundamental arithmetic skills
2. Use of formulas and terms, arithmetic skills of a higher level
3. Mathematical concepts
4. Understanding of texts

In the following I will give examples for the different categories. In every case the specific reasons for their occurrence are discussed and possible ways to remove the mistakes are suggested.

Categories of mistakes

1. Fundamental arithmetic skills

Students carry out the following calculation:

$$(x + y)^5 = x^5 + y^5$$

Apparently the students do not analyze the term. As a result they don't recognize, what is written, and confuse the term with $(x \cdot y)^5 = x^5 y^5$.

Ways out are:

- Training using special exercises:
 - Exercises, which provoke the students to make typical mistakes
 - Terms are varied, which are mixed up usually
- Calculations done with paper and pencil are very recommended.
The mistakes are corrected explicitly one by one and can therefore be recognized better.

2. Wrong usage of mathematical formulas or terms

We consider two different examples.

a) The first mistake occurs in the context of differentiation: The function

$$f(x) = e^{4x}$$

has to be differentiated. As a result is written:

$$f'(x) = 4e^{3x}$$

The reason for this mistakes is signal oriented thinking: Because there is a power, the power rule must be used. Parallel to this the term is not analyzed, so that it is not recognized, which part of the power x is. As a result the differentiation is carried out in a wrong way.

b) The sigma sign is used in a wrong way:

$$\sum_{k=1}^n 4k = \sum_{k=1}^m 4k + \sum_{k=m}^n 4k$$

Apparently the use of the sigma sign is not understood.

Moreover there is a collision with a situation in everyday life: When we say we go on a trip from Göttingen to Hannover and then from Hannover to Berlin, the town Hannover is mentioned twice.

A suitable comparison is one with the situation in an office, where queues have to be formed: One for names beginning with A to K, one for names beginning with L to P and so on. The letter K is mentioned only once.

Ways out are:

- Calculations with paper and pen, which are corrected afterwards
- Some possible solutions are presented and the correct solution has to be identified

This is very successful: The mistakes disappear nearly completely!

3. Misconceptions of fundamental mathematical concepts

a) The concept of functions

There exist general misconceptions concerning the meaning of a function, for instance:

- A function is an equation or something to be calculated.
- A function is somewhat like a trajectory, i.e. a continuous path without kinks - at least not too many.
- There exists no association with a relation.

As a consequence of the misconception of the function the meaning of the equation $f(x) = x^2 + 5$ is not understood. When students want to calculate the second coordinate of the point $P(3/y_P)$, they write $f(x) = 3^2 + 5$.

b) The meaning of and the relation between $f(x)$ and $f'(x)$

The meaning of a function $f(x)$ and their derivative $f'(x)$ is not understood as well the relation between these two functions. The misconceptions are the following:

- general misconception of the meaning of $f'(x)$
- The usual explanations provoke an understanding of $f'(x)$ as the total change of $f(x)$ in an interval.
- The use of different positions for the interpretation of $f(x)$ and $f'(x)$ is difficult, e.g.: When the values of $f'(x)$ increase, it is concluded that $f(x)$ increases, even when the values of $f'(x)$ are negative.

Ways out:

- special teaching, exercises and questions concerning the concepts

The following exercises are recommended:

- Graphs of $f(x)$ and $f'(x)$ have to be matched.
- Correct solutions have to be identified among some possible solutions presented.

4. Understanding of texts

A considerable percentage of students has difficulties to understand mathematical texts. The reasons are the following:

- Special phrases used in context of mathematics are not understood. For example: What is meant by „for every“, „there exists a“ or „almost all“?
- There is a lack of fundamental reading skills.

Ways out:

- exercises and questions which deal with concepts
- A phrase and some possible meanings are presented. The correct meaning has to be identified.
- The students have to read texts concerning well defined topics as homework.

Methods to deal with mistakes

In order to deal successfully with the mistakes and to remove them, the following is recommended:

- identification of categories
- different methods of teaching, i.e.:
 - handgraded homework
 - reading of texts
 - different ways of working and learning:
working by oneself, working in groups, coached groups for students with more problems

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